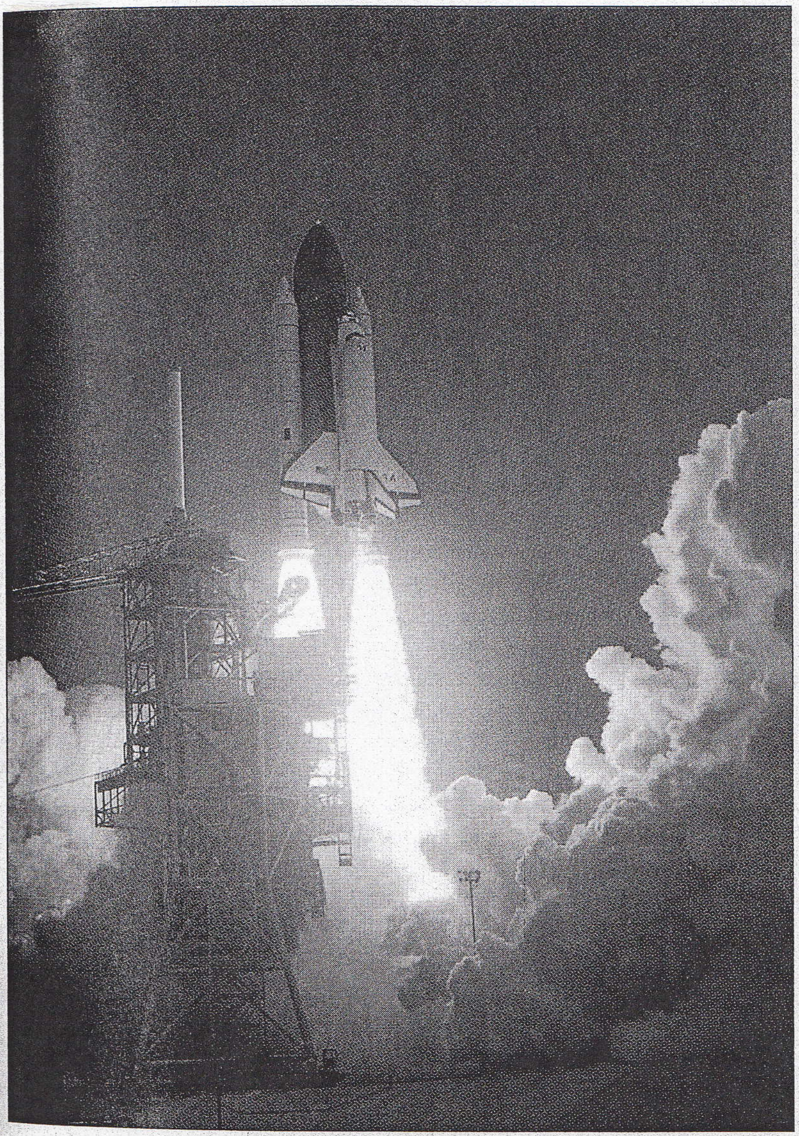
Chapter Two

2

Gravity and Motion



The Space Shuttle blasts off.
(Courtesy of NASA.)

Gravity gives the Universe its structure. It is a universal force that acts on all the objects in the Universe so that every particle is drawn toward every other particle by its pull. Gravity holds together astronomical bodies of all sizes, from the Earth to the Universe itself. But the role of gravity extends beyond giving structure to astronomical bodies. Gravity also controls their motions, holding the Earth in orbit around the Sun, the Sun in orbit around the Milky Way, and the Milky Way within the Local Group. Thus, gravity and motion are tightly connected in the Universe. This connection is the theme of this chapter.

2.1 SOLVING THE PROBLEM OF ASTRONOMICAL MOTION

Astronomers of antiquity did not make the connection between gravity and astronomical motion that we recognize today. The astronomers were puzzled as to why, if the Earth moved, they and it did not simply fly off into space, and they were also mystified about what kept the planets moving.

The solutions to these mysteries began with a series of careful experiments conducted by Galileo in the 1600s. Apart from his famous—and perhaps fictitious—demonstration of weights dropped from the Leaning Tower of Pisa, Galileo experimented with projectiles and with balls rolling down planks. Such experiments led him to propose several laws of motion. More important, perhaps, these experiments demonstrated the power and importance of the experimental method for verifying scientific conjectures.

2.2 INERTIA

Central to Galileo's laws of motion is the concept of **inertia.** Inertia is the tendency of a body at rest to remain at rest and a body in motion to keep moving in a straight line at a constant speed. Aristotle noted that bodies at rest resist being moved, but he failed to link this property to the tendency of objects to keep moving once they are set in motion. Kepler also recognized inertia's importance and in fact was first to use that term. However, Galileo not only proposed this property of matter but also demonstrated it by real experiment.

In one such experiment, Galileo rolled a ball down a sloping board over and over again and noticed that it always sped up as it rolled down the slope (fig. 2.1). He next rolled the ball up a sloping board and noticed that it always slowed down as it

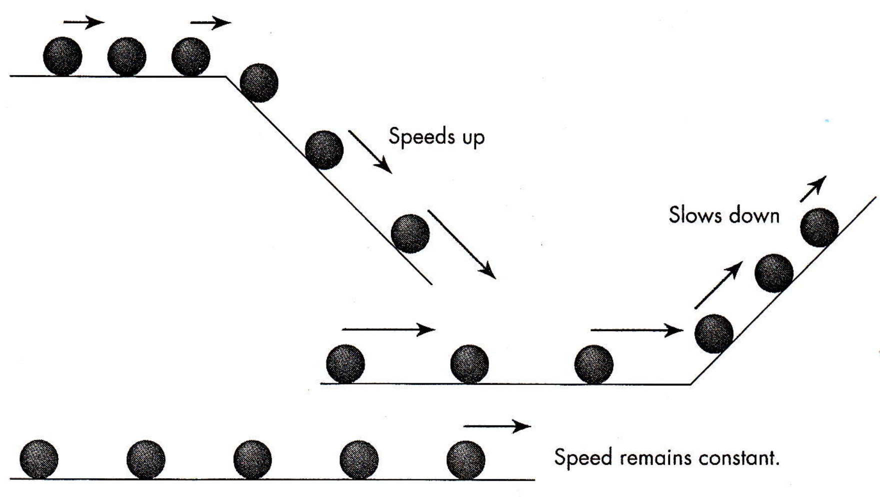


FIGURE 2.1
A ball rolling down a slope speeds up. A ball rolling up a slope slows down. A ball rolling on a flat surface rolls at a constant speed if no forces act on it.

approached the top. He hypothesized that if a ball rolled on a flat surface and no forces—such as friction—acted on it, its speed would neither increase nor decrease but remain constant. That is, in the absence of forces, inertia keeps an object already in motion moving at a fixed speed. Inertia is familiar to us all even in everyday life. Apply the brakes of your car suddenly, and the inertia of the bag of groceries beside you keeps the bag moving forward at its previous speed until it hits the dashboard or spills onto the floor.

Newton recognized the special importance of inertia. He described it in what is now called **Newton's first law of motion** (sometimes referred to simply as the law of inertia). The law can be stated as follows:

A body continues in a state of rest or motion in a straight line at a constant speed unless made to change that state by forces acting on it.

In applying Newton's first law, we should note two important points. First, we have not defined force yet but have relied on our intuitive feeling that a force is anything that pushes or pulls. Second, we need to note that when we use the term *force,* we are talking about *net* force; that is, the total of all forces acting on a body. For example, if a brick at rest is pushed equally by two opposing forces, the forces are balanced. Therefore, the brick experiences no net force and accordingly does not move (fig. 2.2).

Newton's first law may not sound impressive at first, but it carries the idea that is crucial in astronomy, that if a body is *not* moving in a straight line at constant speed, some net force *must* be acting on it.

Actually, Newton was preceded in stating this law by the seventeenth-century Dutch scientist Christiaan Huygens. However, Newton went on to develop additional physical laws and—more important for astronomy—showed how to apply them to the Universe. For example, let us look at what happens if we swing a mass tied to a string in a circle. Newton's law tells us that the mass's inertia will carry it in a straight line if no forces act. What force, then, is acting on the circling mass? The force is the one exerted by the string, preventing the mass from moving in a straight line and keeping it in a circle. We can feel that force as a tug on the string, and we can see its importance if we suddenly let go of the string. With the force no longer acting on it, the mass flies off in a straight line, demonstrating the first law, as illustrated in figure 2.3.

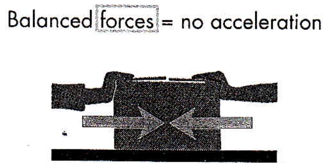


FIGURE 2.2
Balanced forces lead to no acceleration.

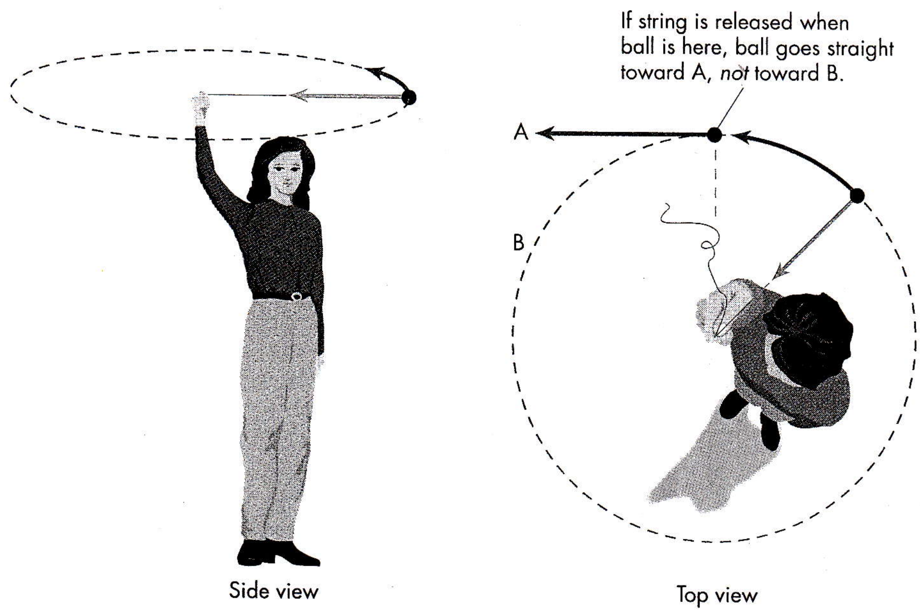


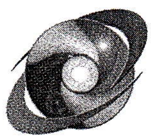
FIGURE 2.3
For a mass on a string to travel in a circle, a force must act along the string to overcome inertia. Without that force, inertia makes the mass move in a straight line.

We can translate this example to an astronomical setting and apply it to the orbit of the Moon around the Earth, or of the Earth around the Sun, or to the Sun in orbit around the Milky Way. Each of these bodies follows a curved path. Therefore, each must have a force acting on it, the origin and nature of which we will now describe.

2.3 ORBITAL MOTION AND GRAVITY

Newton was not the first person to attempt to discover and define the force that holds planets in orbit around the Sun. Nearly 100 years before Newton, Kepler recognized that some force must hold the planets in their orbits and proposed that something similar to magnetism might be responsible. Newton was not even the first person to suggest that gravity was responsible. That honor belongs to Robert Hooke, another Englishman, who noted gravity's role in celestial motions several years before Newton published his law of gravity in 1687. Newton's contribution is nevertheless special because he demonstrated the *properties* that gravity must have if it is to control planetary motion. Moreover, Newton went on to derive equations that describe not only gravity but also its effects on motion. The solution of these equations allowed astronomers to predict the position and motion of the planets and other astronomical bodies.

According to legend, Newton realized gravity's role when he saw an apple falling from a tree. The falling apple drawn downward to the Earth's surface presumably made him speculate whether Earth's gravity might extend to the Moon. Influenced by an apple or not, Newton correctly deduced that Earth's gravity, if weakened by distance, could explain the Moon's motion.



Newton's cannon

Most of Newton's work is highly mathematical, but as part of his discussion of orbital motion, he described a thought experiment to demonstrate how a body can move in orbit. Thought experiments are not actually performed; rather, they serve as a way to think about problems. In Newton's thought experiment, we imagine a cannon on a mountain peak firing a projectile (fig. 2.4A). From our everyday experience, we know that whenever a body is thrown horizontally, gravity pulls it downward so that its path is an arc. Moreover, the faster we throw, the farther the body travels before striking the ground.

We now imagine increasing the projectile speed more and more, allowing it to travel ever farther. However, as the distance traveled becomes very large, we see that the Earth's surface curves away below the projectile (fig. 2.4B). Therefore, if the projectile moves sufficiently fast, the Earth's surface may curve away from it in such a way that the projectile will never hit the ground. Such is the nature of orbital motion and

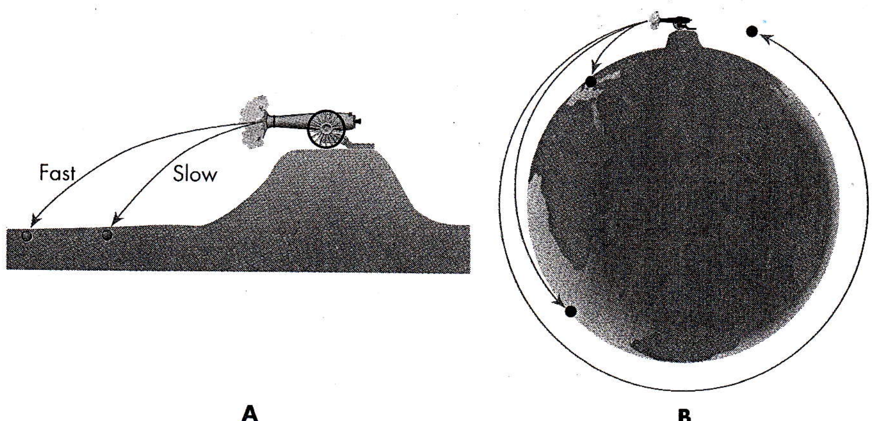


FIGURE 2.4
(A) A cannon on a mountain peak fires a projectile. If the projectile is fired faster, it travels farther before hitting the ground. (B) At a sufficiently high speed, the projectile travels so far that the Earth's surface curves out from under it, and the projectile is in orbit.

how the Moon orbits the Earth. The balance between inertia and the force of gravity maintains the orbit.

We can analyze this thought experiment more specifically with Newton's first law of motion. According to that law, in the absence of forces, the projectile will travel in a straight line at constant speed. But because a force, gravity, is acting on the projectile, its path is not straight but curved. Moreover, the law helps us understand that the projectile does not stop because its inertia carries it forward.

Notice that in the above discussion we used no formulas. All we needed was Newton's first law and the idea that gravity supplies the deflecting force. However, if we are to understand the particulars of orbital motion, we require additional laws. For example, to determine how rapidly the projectile must move to be in orbit, we need laws that have a mathematical formulation.

2.4 NEWTON'S SECOND LAW OF MOTION

We showed that an object's inertia means that it will move at a constant speed in a straight line in the absence of forces. However, suppose forces do act on the object. How much deviation from straight-line motion will such forces produce? To answer that question, we need to define more carefully what we mean by motion.

Motion of an object is a change in its position, which we can characterize in two ways: by the direction of the object and by its speed. For example, a car is moving east at 40 miles per hour. If the car's speed and direction remain constant, we say it is in uniform motion. If the car changes either its speed or direction, it is no longer moving uniformly, as depicted in figure 2.5. Such nonuniform motion is defined as **acceleration.**

Acceleration

We are all familiar with acceleration as a change in speed. For example, when we step on the accelerator in a car and it speeds up from 30 to 40 mph, we say the car is accelerating. Its speed has changed, and its motion is therefore nonuniform. Although in everyday usage acceleration implies an increase in speed, scientifically any change in speed is an acceleration. Thus, technically, a car "accelerates" when we apply the brakes, and it slows down.

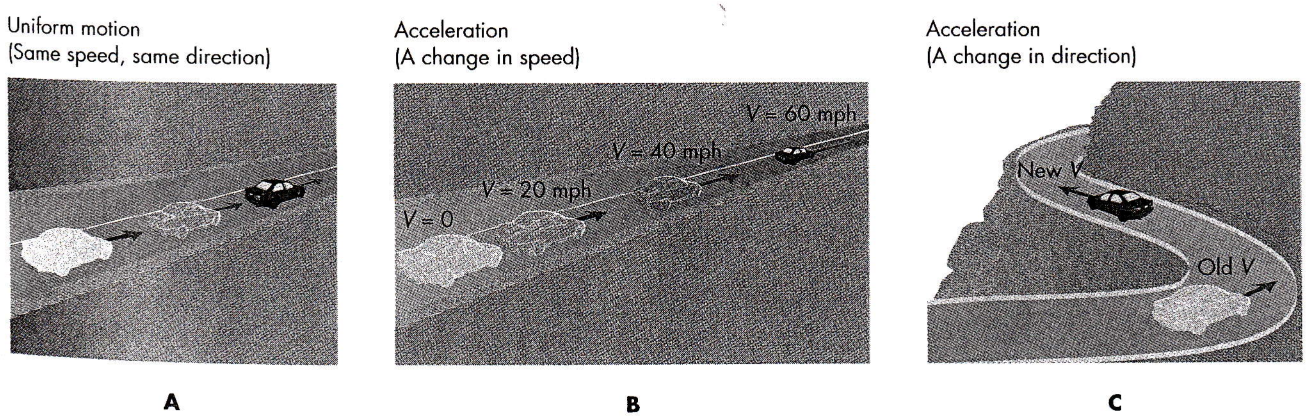


FIGURE 2.5
Views of a car in uniform motion and accelerating. (A) Uniform motion implies no change in speed or direction. The car moves in a straight line at a constant speed. If either an object's (B) speed or (C) direction changes, the object undergoes an acceleration.

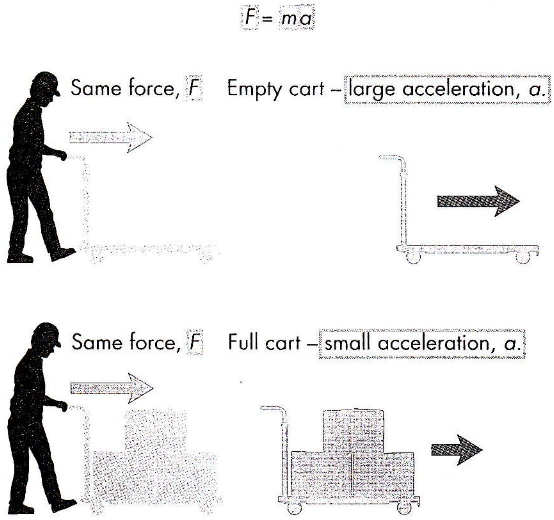


FIGURE 2.6
A loaded cart will not accelerate as easily as an empty cart.

In this example, we produced an acceleration by changing a body's speed.* We can also produce an acceleration by changing a body's direction of motion. For example, suppose we drive a car around a circular track at a steady speed of 30 miles per hour. At each moment, the car's direction of travel is changing, and therefore, it is not in uniform motion. Similarly, a mass swung on a string or a planet orbiting the Sun is experiencing nonuniform motion and is therefore, accelerating. In fact, a body moving in a circular orbit constantly accelerates, even though its speed is not changing.

How do we produce an acceleration? Newton realized that for a body to accelerate, a force must act on it. For example, to accelerate—change the direction of—the mass whirling on a string, we must constantly exert a pull on the string. Similarly, to accelerate a shopping cart, we must exert a force on it. In addition, experiments show that the acceleration we get is proportional to the force we apply. That is, a larger force produces a larger acceleration. For example, if we push a shopping cart gently, its acceleration is slight. If we push harder, its acceleration is greater. But experience shows us that more than just force is at work here. For a given push, the amount of acceleration also depends on how full the cart is. A lightly loaded cart may scoot away under a slight push, but a heavily loaded cart hardly budges, as illustrated in figure 2.6. Thus, the acceleration produced by a given force also depends on the amount of matter being accelerated.

Mass

The amount of matter an object contains is determined by a quantity that scientists call mass. Technically, **mass** measures an object's inertia. The more inertia, the more mass, and vice versa.

Scientists generally measure the mass of ordinary objects in grams, or kilograms (1 kilogram—abbreviated kg—equals 1000 grams). For example, under normal conditions, a liter of water (roughly a quart) has a mass of 1 kilogram, but it is important to remember that mass is not the same as weight. Because an object's measured mass describes the amount of matter in it, its mass in kilograms is a fixed quantity. An object's weight, however, measures the force of gravity on it, a point we will explore more later. Thus, although a body's mass is fixed, its weight changes if the local gravity changes. For example, on Earth we have one weight but on the Moon, where gravity is less, we have a lesser weight, but no matter where we are, we have the same mass.

Mass is the final quantity we need to define before we can understand **Newton's second law of motion** in its full form. Mathematically, the law states that

$$F = ma$$

F = Any force
m = Mass of the body being accelerated
a = Amount of acceleration

or, in words:

The amount of acceleration (a) that a force (F) can produce depends on the mass (m) of the object being accelerated.

This astonishingly simple equation allows scientists to predict virtually all features of a body's motion. With $F = ma$ and with knowledge of the masses and the forces in action, engineers and scientists can, for example, drop a spacecraft safely between Saturn and its rings or use a computer to design an airplane that will fly successfully without being tested.

*In our discussion, we use the word *speed* to denote the rate of motion, irrespective of its direction. Were we to be more technically correct, we would use the term *velocity*, which means the speed in a *given direction*. Thus, a body's velocity changes if *either* its speed or direction changes. With velocity so defined, acceleration is simply a change in velocity.

2.5 THE LAW OF GRAVITY

Using Newton's second law, we can now determine an object's motion if we know its initial state of motion and the forces acting on it. For astronomical bodies, that force is often limited to gravity, and so to predict their motion, we need to know how to calculate gravity's force. Once again we encounter Newton's work, for it was he who first worked out the **law of gravity.** On the basis of his study of the Moon's motion, Newton concluded the following:

Every mass exerts a force of attraction on every other mass. The strength of the force is directly proportional to the product of the masses divided by the square of their separation.

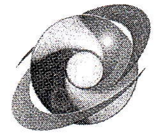
We can write this extremely important result in a shorthand mathematical manner as follows:

Let m and M be the masses of the two bodies (fig. 2.7) and let the separation between their centers be r. Then the strength of the force between them, F, is

$$F = \frac{GMm}{r^2}$$

The factor G is a constant whose value is found by measuring the force between two bodies of known mass and separation. The resulting number for G depends on the units chosen to measure M, m, r, and F, but once determined, G is the same as long as the same units are used. For example, if M and m are measured in kilograms, r in meters, and F in SI units,* then $G = 6.67 \times 10^{-11}$ meters^3/(kilogram-second^2) [m^3-kg^{-1}-s^{-2}].

Writing the law of gravity as an equation helps us see several important points. If either M or m increases and the other factors remain the same, the force increases. If r (the distance between two objects) increases, the force gets weaker. Moreover, the force weakens as the square of the distance. That is, if the distance between two masses is doubled, the gravitational force between them decreases by a factor of four, not two. Finally, although one body's gravitational force on another weakens with increasing distance, the gravitational force never completely disappears. Thus, the gravitational attraction of a body reaches across the entire Universe, so the Earth's gravity not only holds you on to its surface but also extends to the Moon and exerts the force that holds the Moon in orbit around the Earth.



Gravity produces a force of attraction between bodies

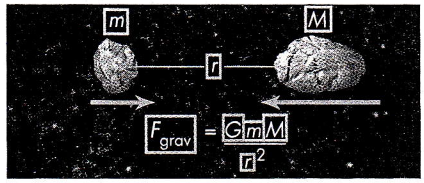


FIGURE 2.7
Gravity produces a force of attraction between bodies. The strength of the force depends on the product of their masses, m and M, and the square of their separation, r. G is the universal gravitational constant.

F = Strength of the force between two bodies
M = Mass of one body
m = Mass of second body
r = Separation between the bodies' centers
G = Constant

2.6 NEWTON'S THIRD LAW OF MOTION

Newton's studies of motion and gravity led him to yet another critical law, which relates the forces that bodies exert on each other. This additional relation, **Newton's third law of motion,** is sometimes called the law of action-reaction. This law states:

When two bodies interact, they create equal and opposite forces on each other.

Two skateboarders side by side may serve as a simple example of the third law (fig. 2.8). If A pushes on B, *both* move. According to Newton's law, when A exerts a force on B, B exerts a force on A so that both are accelerated.

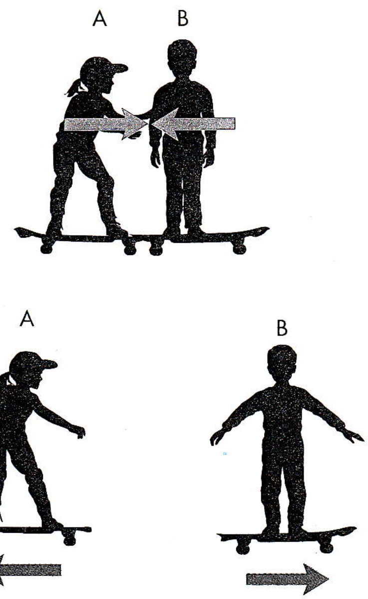


FIGURE 2.8
Skateboarders illustrate Newton's third law of motion. When A pushes on B, an equal push is given to A by B.

*SI units (System International) use the kilogram, meter, and second as the fundamental units.

The gravitational force between the Earth and the Sun affords an astronomical example of Newton's third law and at the same time leads us a step closer to understanding orbital motion. The gravitational force of the Earth on the Sun is exactly equal to the gravitational force of the Sun on the Earth. We can see this perhaps surprising result from Newton's law of gravity where the gravitational force between two bodies depends on the product of their masses. We thus get the same force regardless of whether we let the Earth act on the Sun or vice versa. Why, then, does the Earth orbit the Sun and not the other way around?

The second law supplies the answer. If we translate $F = ma$ into $a = F/m$, we see that the acceleration a body feels is inversely proportional to its mass; that is, the more massive it is, the more force is required to accelerate the mass. Thus, even though the forces acting on the Earth and Sun are precisely equal, the Sun accelerates 300,000 times less because it is 300,000 times more massive than the Earth. Thus, because the Earth's acceleration is so much larger than the Sun's, the Earth does most of the moving. In fact, however, the Sun does move a little bit as the Earth orbits it, much as you must move if you swing a child around you in play.

2.7 MEASURING A BODY'S MASS USING ORBITAL MOTION

Knowledge of orbital motion is important for more than just understanding the paths of astronomical objects. From the orbit's properties (such as size and orbital period), astronomers can deduce physical properties of the orbiting objects, such as their mass.

The basic method for determining an astronomical object's mass uses a modified form of Kepler's Third Law and was first worked out by Newton using his laws of motion and gravity. The underlying idea is very simple: the masses of the orbiting bodies determine the gravitational force between them. The gravitational force in turn sets the properties of the orbit. Thus, from knowledge of the orbit, astronomers can work backward to find the masses of the objects. To see how this can be done, we consider a very simple case: orbital motion in a perfect circle with the orbiting body having a mass so small it can be ignored compared with that of the central body. These restrictions are met to high precision in many astronomical systems, such as the Earth's motion around the Sun and the Sun's motion around the Milky Way. By assuming that the mass of the central body is large compared with the orbiting body, we can ignore the acceleration of the central body (as we just discussed above) and assume it is at rest. These assumptions simplify the problem but are not essential for its solution.

To work out the orbital properties of a body moving around another, we use Newton's laws of motion and his law of gravity. From the first law, we know that if a body moves along a circular path, there is a net force (an unbalanced force) acting on it because balanced forces give straight-line motion. This force* must be applied to any body moving in a circle, whether it is a car rounding a curve, a mass swung on a string, or the Earth orbiting the Sun.

Using Newton's second law of motion and some algebra gives us an equation that shows that if a mass (m) moves with a velocity (V) around a circle of radius (r), the force needed to hold it in a circular orbit is

F = Force needed to hold a body in orbit
m = Mass of the body
r = Radius of the orbit
V = Velocity of the body

$$F = \frac{mV^2}{r}$$

Using the above equation, we can find the orbital velocity of a planet around the Sun. Let the Sun's mass be M and the planet's mass be m, the latter of which is assumed

*This force is called a "centripetal force," which is the force applied to draw an object *inward* toward the center of the orbit.

to be much smaller than M. Assume the planet moves in a circular orbit of radius (r) at a velocity (V) so that the force required to hold the planet in orbit is mV^2/r. That force—the force that deflects the planet from its tendency to move in a straight line—is supplied by the gravitational force between the Sun and the planet. We have already defined that force as GMm/r^2.

Because mV^2/r is the force required to hold the planet in orbit, it must equal the force of gravity.

$$\frac{mV^2}{r} = \frac{GMm}{r^2}$$

We can cancel m out, and one of the r's to obtain

$$V^2 = \frac{GM}{r}$$

Finally, we take the square root of both sides to obtain the orbital velocity.

$$V = \sqrt{\frac{GM}{r}}$$

The above equation, giving the orbital velocity in terms of M and r, can be used to determine the mass of the central body if the orbiting object's velocity and distance from it are known.

EXTENDING OUR REACH

WEIGHING THE SUN

To find the mass (M) of the Sun, we go back to our equation for V^2, cross-multiply by r, and divide by G to obtain

$$M = \frac{V^2 r}{G}$$

Therefore, to evaluate M, we need V, the Earth's orbital velocity (the speed with which it is moving in its orbit). We find V by dividing the circumference of its orbit, C, by the time it takes the Earth to complete one orbit, a length of time we call the orbital period, P. Thus

$$V = \frac{C}{P}$$

The formula for the circumference of a circle is $C = 2\pi r$. Thus

$$V = \frac{2\pi r}{P}$$

Putting this expression for V into the equation for M, above, gives

$$M = \left(\frac{2\pi r}{P}\right)^2 \frac{r}{G}$$

$$= \frac{4\pi^2 r^3}{GP^2}$$

This is a modified form of Kepler's Third Law and it is important because we can use it to measure the mass of an object given the orbital period, P, and orbital radius, r, of a much smaller object moving around it. For example, suppose we want to find the Sun's mass, M.

According to the expression for M, we need P and r for the Earth's orbit. If we measure P in seconds and r in meters, then the gravitational constant $G = 6.67 \times 10^{-11}$ m^3-kg^{-1}-s^{-2}.

To find P in seconds, we remember that it takes the Earth one year to orbit the Sun. Thus, P is one year. We can express P in seconds by multiplying the number of seconds in a minute (60), times the number of minutes in an hour (60), times the number of hours in a day (24), times the number of days in a year (365.25). The result of that calculation, rounded off to three significant figures, is

$$P = 3.16 \times 10^7 \text{ seconds}$$

Similarly, we need r, the Earth–Sun distance, in meters, which we can look up in the appendix and find is 1.50×10^{11} m. Putting these values and the value of π and G into the expression for M we find

$$M = \frac{4(3.14)^2 \times (1.50 \times 10^{11}\ \text{m})^3}{6.67 \times 10^{-11}\ \text{m}^3\text{-kg}^{-1}\text{-s}^{-2} \times (3.16 \times 10^7\ \text{s})^2}$$

$$= \frac{4 \times 9.86 \times 3.38 \times 10^{33}\ \cancel{\text{m}^3}}{6.67 \times 9.99 \times 10^{14-11}\ \cancel{\text{m}^3}\text{-kg}^{-1}\cancel{\text{s}^{-2}}\cancel{\text{s}^2}}$$

$$= 2 \times 10^{30}\ \text{kg}$$

The same method can be used to find the mass of any body around which another object orbits. Thus, gravity becomes a tool for determining the mass of astronomical bodies, and we shall use this method many times throughout our study of the Universe.

There are other ways we can use the law of gravity. For example, we can use it to find out how much we would weigh on another planet and how fast a spacecraft must move to escape from a planet's surface.

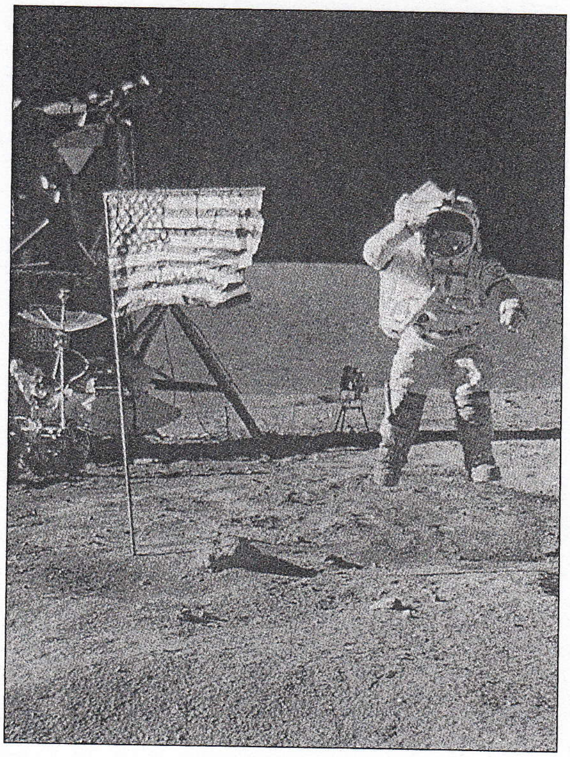
2.8 SURFACE GRAVITY

Surface gravity measures the gravitational attraction at a planet's or star's surface. It is the *acceleration* on a mass created by the local gravitational force, not the force itself. This acceleration determines how fast objects fall. To understand the importance of surface gravity, recall that mass measures the amount of material an object contains and is therefore constant. On the other hand, a body's weight depends on its mass and the acceleration of gravity. Thus, surface gravity determines what a mass weighs. In addition to determining weight on a planet, surface gravity also influences a planet's shape and whether it has an atmosphere. For example, small bodies such as asteroids are not spherical because their surface gravity is too weak to crush them into round shapes. Likewise, a small surface gravity makes it hard for a planet to keep an atmosphere.

We determine the strength of a planet's surface gravity as follows.

The law of gravity states that a planet of mass M exerts a gravitational force F on a body of mass m at a distance r from its *center* given by

$$F = \frac{GMm}{r^2}$$



An astronaut can make huge leaps in the Moon's low gravity.
(Courtesy of NASA.)

At the planet's surface, $r = R$, the planet's radius, so

$$F = \frac{GMm}{R^2}$$

Newton's second law ($F = ma$) tells us that for any force, F, the acceleration it produces on a body of mass m is $a = F/m$. Therefore,

$$ma = \frac{GMm}{R^2}$$

Canceling out the m's then gives

$$a = \frac{GM}{R^2}$$

Thus, a planet's surface gravity depends on its mass and radius. That dependence is such that two planets with the same radius but different masses will have different surface gravities. In particular, if two planets have the same radius but different masses, the planet with the larger mass has the larger surface gravity. Similarly, if two planets have the same mass but different radii, the planet with the larger radius has less surface gravity.

Surface gravity is usually denoted by the letter g (the origin of the phrase "Pulling g's," which is used by pilots). We can therefore write that

$$g = \frac{GM}{R^2}$$

g = Surface gravity
G = Constant
M = Mass of the attracting body
R = Radius of the attracting body

Because the surface gravity depends on the mass and radius of the attracting body, the strength of the surface gravity is different from body to body. For example, we can compare the surface gravity of the Moon with that of the Earth to show why you would weigh less on the Moon than on the Earth.

To make the calculation, we need to know the mass and radius of the Earth and the Moon. Those numbers are given in table 2.1.

Because the Earth has greater mass, we might guess that its surface gravity, g, is greater than the Moon's. However, the Moon's radius is smaller, so its surface gravity might be greater. Thus, to determine which body has the larger surface gravity, we need to evaluate g mathematically.

If we put numbers into the equation for surface gravity, we find that

$$g_{\text{Earth}} = \frac{GM}{R^2}$$

$$g_{\text{Earth}} = \frac{6.7 \times 10^{-11}\ \text{m}^3\text{-s}^{-2}\text{-kg}^{-1} \times 6.0 \times 10^{24}\ \text{kg}}{(6.4 \times 10^6\ \text{m})^2} = 9.8\ \text{m-s}^{-2}$$

and

$$g_{\text{Moon}} = \frac{6.7 \times 10^{-11}\ \text{m}^3\text{-s}^{-2}\text{-kg}^{-1} \times 7.3 \times 10^{22}\ \text{kg}}{(1.7 \times 10^6\ \text{m})^2} = 1.7\ \text{m-s}^{-2}$$

TABLE 2.1
Mass and Radius of the Earth and Moon*

	Mass	Radius
Earth	6.0×10^{24} kg	6.4×10^6 meters
Moon	7.3×10^{22} kg	1.7×10^6 meters
Ratio Earth/Moon	81	3.8

*Numbers have been rounded slightly.

Thus, the ratio of g on the Earth to g on the Moon is about 6 : 1 so that you weigh about 6 times more on the Earth than you would on the Moon. That fact allowed the astronauts to make such huge leaps on the Moon.

You should notice that the calculation could have been made more quickly by taking the ratio of the g's directly. In that case, we would have

$$\frac{g_{\text{Earth}}}{g_{\text{Moon}}} = \frac{GM_{\text{Earth}}/(R_{\text{Earth}})^2}{GM_{\text{Moon}}/(R_{\text{Moon}})^2}$$

$$= \frac{M_{\text{Earth}}/M_{\text{Moon}}}{(R_{\text{Earth}}/R_{\text{Moon}})^2}$$

$$= \frac{(81)}{(3.8)^2} = 5.6 \sim 6$$

To illustrate this point once more, let's compare our weight on the Earth to our weight on Jupiter (assuming we could stand on Jupiter). Jupiter's mass is about 300 times that of the Earth. Its radius is about 10 times larger than the Earth's. The mass ratio contributes a factor of 300 toward a greater g on Jupiter, but its larger radius when squared reduces g by a factor of 100, yielding a net change in g of $300/100 = 3$. Thus, the surface gravity on Jupiter is 3 times that of the Earth, and a 150-pound person on Earth would weigh about 450 pounds on Jupiter. By the same reasoning, a 10-ton rocket on Earth would weigh 30 tons on Jupiter. If we wish to explore such massive bodies, how do we overcome their immense gravity?

2.9 ESCAPE VELOCITY

To overcome a planet's gravitational force and escape into space, a rocket must achieve a critical speed known as the **escape velocity.** Escape velocity is the speed an object needs to move away from a body and not be drawn back by its gravity. We can understand how such a speed might exist if we think about throwing an object into the air. The faster the object is tossed upward, the higher it goes and the longer it takes to fall back. Escape velocity is the speed an object needs so that it will never fall back, as depicted in figure 2.9. Thus, escape velocity is of great importance in space travel if craft are to move away from one body and not be drawn back to it. However, escape velocity is also important in many astronomical phenomena, such as whether a planet has an atmosphere and the nature of black holes.*

The escape velocity, V_{esc}, for a spherical body such as a planet or star, can be found from the law of gravity and Newton's laws of motion and is given by the following formula:

V_{esc} = Escape velocity
G = Constant
M = Mass of the body to be escaped from
R = Radius of the body to be escaped from

$$V_{esc} = \sqrt{2GM/R}$$

Here, M stands for the mass of the body from which we are attempting to escape, and R is its radius, as shown in figure 2.10.

Notice in the equation for V_{esc} that if two bodies of the same radius are compared, the larger mass will have the larger escape velocity. Likewise, if two bodies of the same mass are compared, the one with the smaller radius will have the greater escape velocity. We will see in chapter 14 that the huge escape velocity of a black hole arises from its abnormally small radius.

*In chapter 14, we will see that a black hole is an object whose escape velocity equals the speed of light. Thus, light cannot escape from it, thereby making it black.

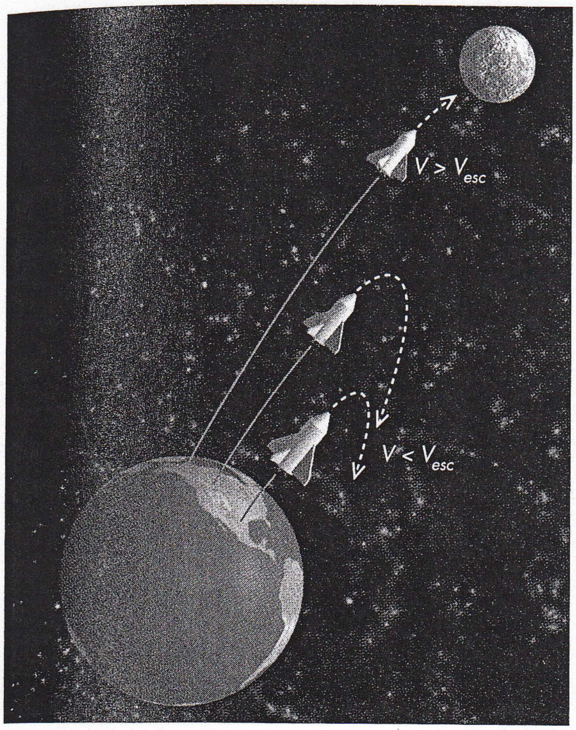


FIGURE 2.9
Escape velocity is the speed an object must have to overcome the gravitational force of a planet or star and not fall back.

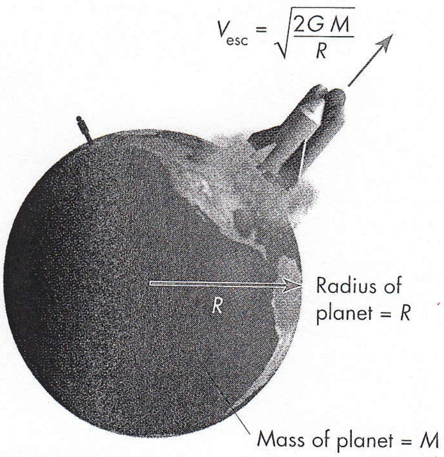


FIGURE 2.10
Calculating the escape velocity from a body.

To illustrate the use of the formula, we calculate the escape velocity from the Moon. From the data in the table 2.1, we find the Moon's radius and mass. We insert these values in the formula for escape velocity and find

$$V_{esc}\,(\text{Moon}) = \sqrt{2GM/R}$$

$$V_{esc}\,(\text{Moon}) = \sqrt{2 \times 6.7 \times 10^{-11}\ \text{m}^3\text{-s}^{-2}\text{-kg}^{-1} \times 7.3 \times 10^{22}\ \text{kg}/1.7 \times 10^{6}\ \text{m}}$$

$$= 2.4 \times 10^{3}\ \text{m/s} = 2.4\ \text{km/s}$$

A similar calculation shows that the escape velocity from the Earth is 11 kilometers per second. Thus, it is much easier to blast a rocket off the Moon than the Earth.* In chapter 6, we will see that this low escape velocity is partly responsible for the Moon's lack of an atmosphere.

Summary

A gravitational force exists between any two objects in the Universe. The strength of this force depends on the masses of the bodies and their separation. Gravitational forces hold astronomical bodies together and in orbit about one another.

A body's inertia makes it remain at rest or move in a straight line at a constant speed unless the body is acted on by a net force. Thus, for a planet to orbit the Sun, the Sun's force of gravity must act on it. The law of inertia and Newton's other laws of motion, when combined with the law of gravity, allow us to relate the size and speed of orbital motion to the mass of the central body.

The gravitational force exerted by a planet determines its surface gravity and escape velocity. The former determines your weight on a planet. The latter is the speed necessary to leave the surface and escape without falling back.

Questions for Review

1. What is meant by inertia?
2. What does Newton's first law of motion tell you about the difference between motion in a straight line and motion along a curve?
3. What is Newton's law of gravity?
4. How does mass differ from weight?
5. If your mass is 70 kilograms on Earth, what is it on the Moon?
6. If you weigh 110 pounds on the Earth, do you weigh 110 pounds on the Moon? Why?
7. What does surface gravity measure?
8. What is meant by escape velocity?

Thought Questions

1. Which do you think has more inertia: a small, inflated balloon or a cinder block? If each were moving toward you at 1 meter/second, which would be easier to catch?
2. A cinder block can be weightless in space. Would you want to kick it with your bare foot? Even if it is weightless, does it have mass?
3. In some amusement park rides, you are spun in a cylinder and are pressed against the wall as a result of the spin. People sometimes describe that effect as being due to "centrifugal force." What is really holding you against the wall of the spinning cylinder? Drawing a sketch and using Newton's first law may help you answer the question.

Problems

1. How many times greater is the Earth's gravitational force on the Moon than the Moon's gravitational force on the Earth? Think about Newton's third law of motion before answering this.
2. Calculate the escape velocity from the Earth, given that the mass of the Earth is 6×10^{24} kilograms and its radius 6×10^{6} meters. In this problem, round off G to 7×10^{-11} meters^3/(kg-s^2).
3. Calculate the escape velocity from the Sun, given that its mass is 2×10^{30} kg and its radius is 7×10^{8} meters.
4. Which body has a larger escape velocity, Mars or Saturn?
   $M_{\text{Mars}} = 0.1\, M_{\text{Earth}}$
   $M_{\text{Saturn}} = 95\, M_{\text{Earth}}$
   $R_{\text{Mars}} = 0.5\, R_{\text{Earth}}$
   $R_{\text{Saturn}} = 9.4\, R_{\text{Earth}}$
5. Calculate the ratio of the escape velocities from the Moon and Earth.

*If an object's mass and radius are given in units of the Earth's, then $V_{esc} \approx 11\sqrt{M/R}$ km/s.

6. Calculate your weight on the Moon.
7. Given that Jupiter is about 5 times farther from the Sun than the Earth, calculate its orbital velocity. How many years does it take Jupiter to complete an orbit around the Sun?
8. Given that the mass of the Milky Way galaxy is 10^{11} times that of the Sun and that the Sun is 2.6×10^{20} meters from its center, what is the Sun's orbital speed around the center of the galaxy? How long does it take the Sun to orbit the Milky Way? (In this problem, we assume that the galaxy can be treated as a single spherical blob of matter. Strictly speaking, this isn't correct, but the far more elaborate math needed to calculate the problem properly ends up giving almost the same answer.)
9. A good baseball pitcher can throw a ball at 100 miles/hr (about 45 meters/s). If the pitcher were on the fictional asteroid Cochise, could the pitcher throw the ball fast enough so it would escape from Cochise? (Note: Assume Cochise is a sphere and that its mass is $M = 9.6 \times 10^{16}$ kg and its radius is 20 km = 2.0×10^4 meters.)
10. Use the modified form of Kepler's third law to find the mass of the imaginary star 57 Fungaloid, given that a planet is in a circular orbit around it at a distance of 3×10^{11} meters with an orbital period of 3 years. Divide your answer by the Sun's mass to see how much more (less) massive the star is than our Sun.

Test Yourself

1. Which of the following demonstrate the property of inertia?
   (a) A car skidding on a slippery road
   (b) The oil tanker *Exxon Valdez* running aground
   (c) A brick sitting on a table top
   (d) Whipping a table cloth out from under the dishes set on a table
   (e) All of the above
2. If an object moves along a curved path at a constant speed, you can infer that
   (a) a force is acting on it.
   (b) it is accelerating.
   (c) it is in uniform motion.
   (d) both (a) and (b) are true.
   (e) neither (a) nor (b) is true.
3. If the distance between two bodies is quadrupled, the gravitational force between them is
   (a) increased by a factor of 4.
   (b) decreased by a factor of 4.
   (c) decreased by a factor of 8.
   (d) decreased by a factor of 16.
   (e) decreased by a factor of 64.
4. The gravitational force exerted by the Sun on the Earth is the same as the gravitational force exerted by the Earth on the Sun.
   (a) True
   (b) False
5. Two planets have identical diameters but differ in mass by a factor of 25. The more massive planet therefore has an escape velocity
   (a) 25 times larger than the other.
   (b) 25 times smaller than the other.
   (c) 5 times larger than the other.
   (d) 625 times larger than the other.
   (e) 50 times larger than the other.

Further Explorations

Duzen, Carl, Jane Nelson, and Jim Nelson. "Classifying Motion." *Physics Teacher* 30 (October 1992): 414.

The following books discuss the laws of motion in greater detail:

Casper, Barry M., and Richard J. Noer. *Revolutions in Physics.* New York: W. W. Norton, 1972.

Hecht, Eugene. *Physics in Perspective.* Reading, Mass.: Addison-Wesley, 1980.

Hewitt, Paul G. *Conceptual Physics.* New York: HarperCollins, 1992.

Kirkpatrick, Larry D., and Gerald F. Wheeler. *Physics: A World View.* Philadelphia: Saunders, 1992.

Web Site

Please visit the *Explorations* web site at **http://www.mhhe.com/arny** for additional on-line resources on these topics.

Key Terms